The Isabelle Refinement Framework

Peter Lammich

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Introduction

- Peter Lammich
    - new assistant professor in FMT group
        - previously in Münster, Munich, Virginia Tech, Manchester
    - research: software verification

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  - research: software verification
- if I'm not working: you'll probably find me rock-climbing
  - but I also enjoy hiking, biking (mtb, road, trek), racket sports (squash, badminton), ...

The Sloth, HVS 5a, at the Roaches in Peak District

Bull's Crack, HVS 5a, at Heptonstall

Sport Climbing (somewhere in the Peaks)

Mountainbiking (at Lake Garda, after TransAlp)

Hiking in the Alps

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- This talk: towards faster verified algorithms at manageable effort

Introduction

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```
procedure AUGMENT($g, f, p$)
    $c_p \leftarrow \min\{g_f(u, v) \mid (u, v) \in p\}$
    for all $(u, v) \in p$ do
        if $(u, v) \in g$ then $f(u, v) \leftarrow f(u, v) + c_p$
        else $f(v, u) \leftarrow f(v, u) - c_p$
    return $f$
procedure EDMONDS-KARP($g, s, t$)
    $f \leftarrow \lambda(u, v).\ 0$
    while exists augmenting path in $g_f$ do
        $p \leftarrow$ shortest augmenting path
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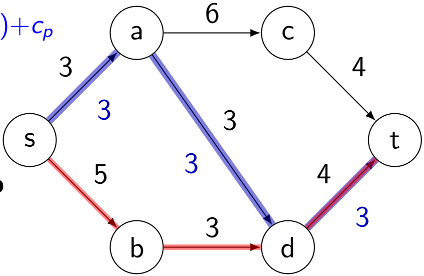

g: flow network s, t: source, target g_f: residual network

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Theorem (Ford-Fulkerson)

For a flow network g and flow f, the following 3 statements are equivalent

1. f *is a maximum flow*
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using basic concepts such as numbers, sets, and graphs. □

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Let δ_f be the length of a shortest s, t - path in g_f.
When augmenting with a shortest path,

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using lemmas about graphs and shortest paths. □

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    - mature, production quality IDE, based on JEdit

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   int flow = 0;
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   int new_flow;

   while (new_flow = bfs(s, t, parent)) {
      flow += new_flow;
      int cur = t;
      while (cur != s) {
         int prev = parent[cur];
         capacity[prev][cur] -= new_flow;
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textbook proof typically covers abstract algorithm.

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- Implementations used for different parts must fit together!

Refinement Architecture (simplified)

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shortest-path-spec

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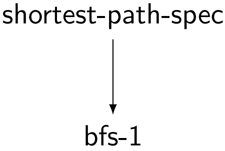

shortest-path-spec

bfs-1

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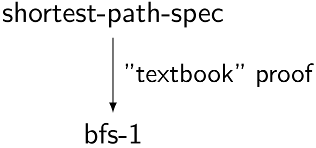

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"textbook" proof

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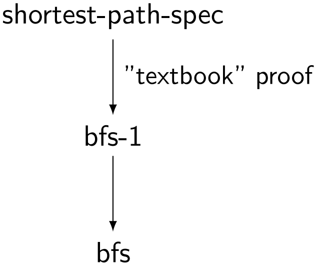

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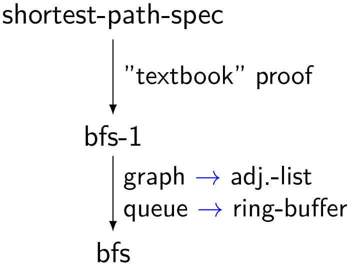

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graph → adj.-list

queue → ring-buffer

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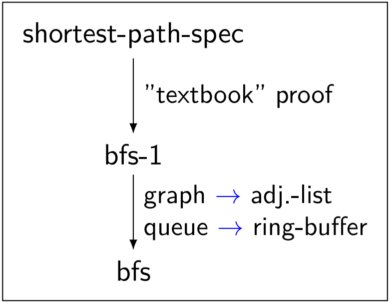

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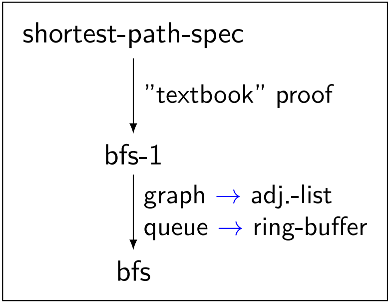

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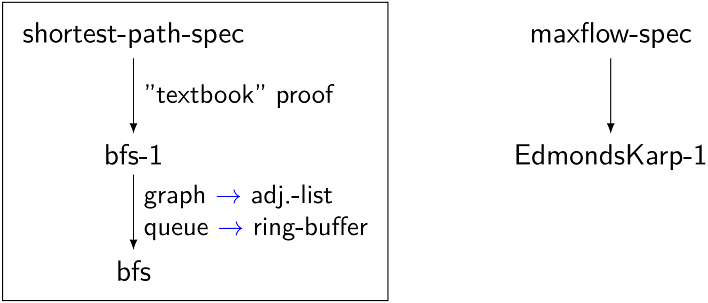

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EdmondsKarp-1

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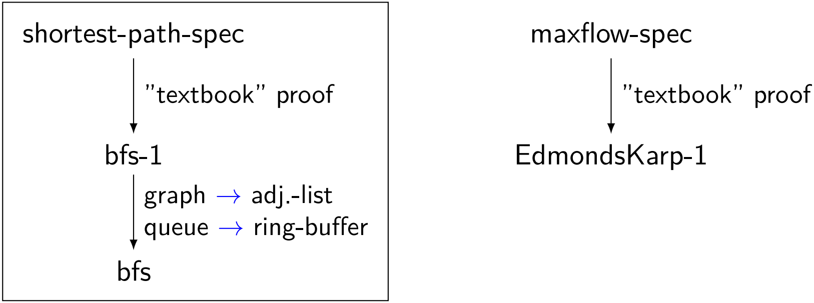

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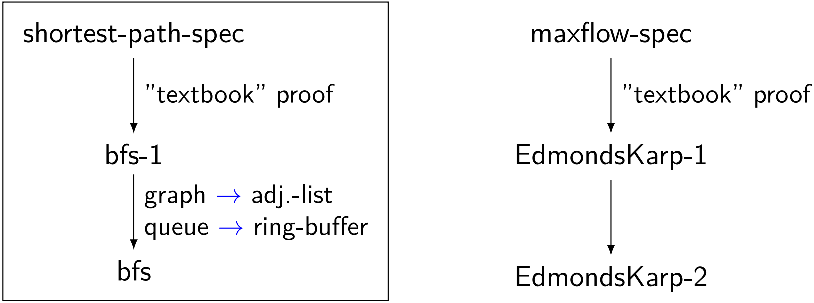

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EdmondsKarp-1

EdmondsKarp-2

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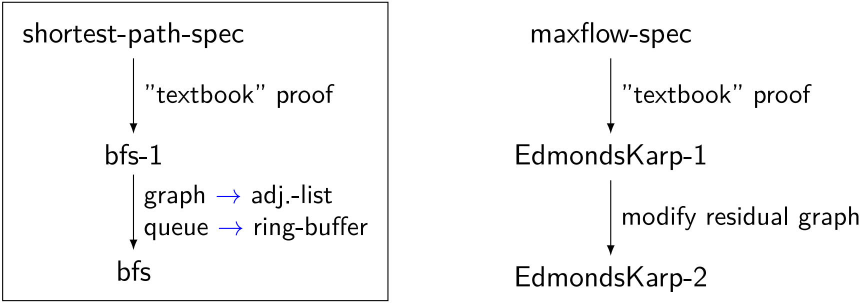

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EdmondsKarp-1

modify residual graph

EdmondsKarp-2

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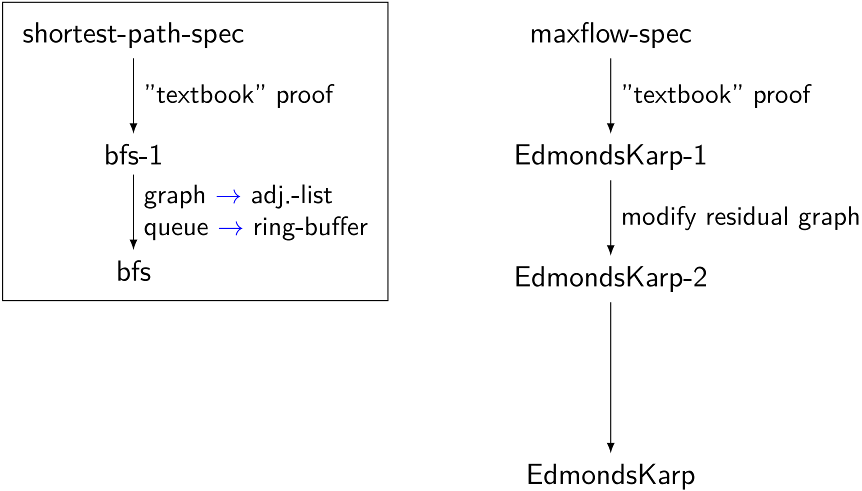

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EdmondsKarp

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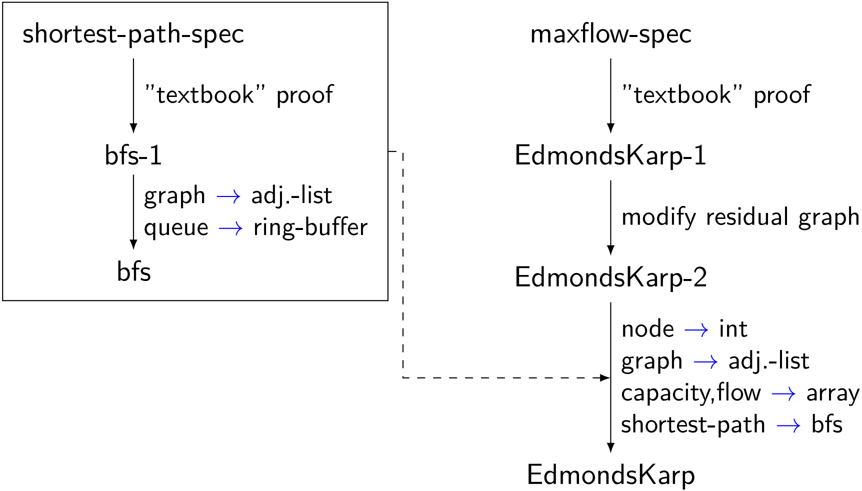

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EdmondsKarp-1

modify residual graph

EdmondsKarp-2

node → int
graph → adj.-list
capacity,flow → array
shortest-path → bfs

EdmondsKarp

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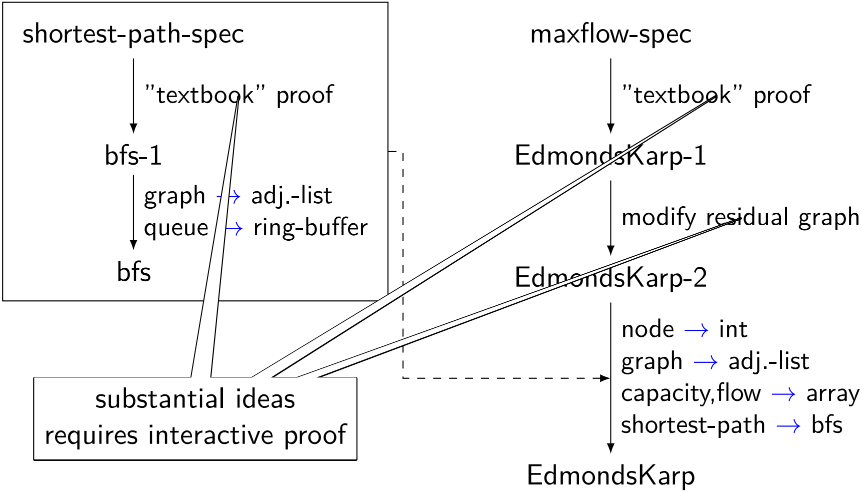

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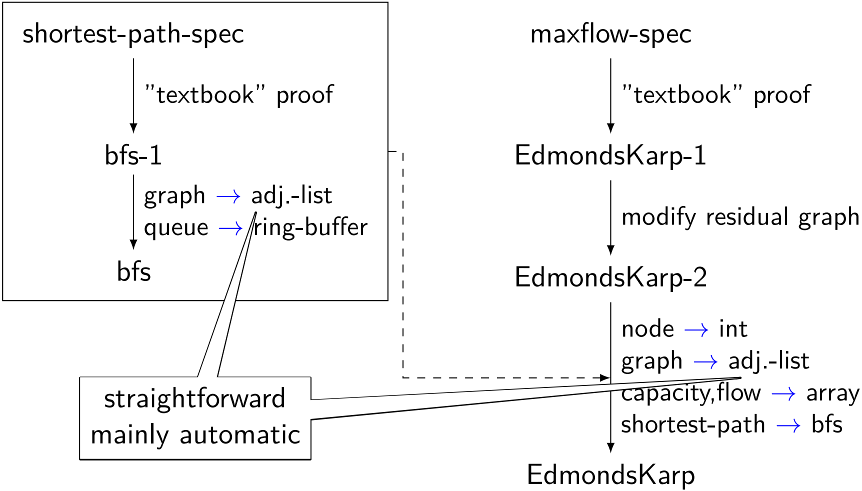
substantial ideas
requires interactive proof

Refinement Architecture (simplified)



shortest-path-spec

"textbook" proof

bfs-1

graph → adj.-list
queue → ring-buffer

bfs

maxflow-spec

"textbook" proof

EdmondsKarp-1

modify residual graph

EdmondsKarp-2

node → int
graph → adj.-list
capacity,flow → array
shortest-path → bfs

EdmondsKarp

straightforward
mainly automatic

The Isabelle Refinement Framework

- Formalization of Refinement in Isabelle/HOL

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- Down to Ocaml/Haskell/Scala/SML and LLVM

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  - As monad

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- Refinement Calculus for Program and Data Refinement
- Automation: VCG, semi-automatic data refinement

Imperative-HOL Backend

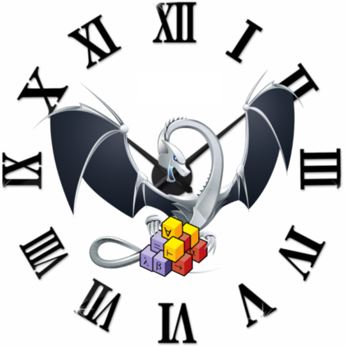
- imperative + functional language
- code generation to Ocaml/Haskell/Scala/SML
- automatic refinement of functional to imperative DS
    - if used linearly

Isabelle-LLVM Backend

- only imperative + bounded integers
- automatic placement of destructors
- semi-automatic in-bound proofs (eg for int → int64)

Refinement with Time

- Prove correctness and complexity
- *Resource currencies* to structure complexity proofs along refinement
- Down to Imperative-HOL / LLVM



Libraries

- Functional and Imperative data structures
    - readily usable for your developments
- Functional:
    - hashtable, red-black-trees, tries, Finger-Trees, (Skew) binomial queues, ...
- Imperative:
    - dynarray, heap, matrix, linked-list, hashtable, bit-vector, union-find, ROBDDs, B-Trees, ...

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- GRAT: SAT-Solver verification tool
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- Introsort + Pdqsort
  - verified correctness and complexity
  - on par with C++ impls from GNU libstdc++ and Boost

Future Work

- Concurrency
- Consolidate frameworks and tools
- Interesting algorithms to verify

Conclusions

Isabelle Refinement Framework

powerful interactive theorem prover
+ stepwise refinement
+ libraries for standard DS
+ lot's of automation
+ efficient backend (LLVM)
= verified and efficient algorithms, at manageable effort